ANALYSIS METHOD FOR TRANSIENT FIELDS IN PLANAR STRUCTURES BY MARCHING-ON-IN-TIME INTEGRAL EQUATION TECHNIQUE

Natalia Georgieva and Eikichi Yamashita
University of Electro-Communications
1-5-1 Chofugaoka, Chofu-shi, Tokyo 182, Japan

ABSTRACT

A new numerical algorithm for the analysis of transient electromagnetic fields in planar structures is proposed based on the magnetic field integral equation (MFIE) and marching-on-in-time approach. This algorithm solves the problem of radiation boundary conditions in a natural way unlike the FDTD method which needs an approximated radiation boundary condition imposed at outer boundaries of the structure. The algorithm is applicable to multilayered planar structures and is competitive to the FDTD method especially in the case of open and radiating problems. The MFIE is applied in combination with boundary element approach and point matching technique.

INTRODUCTION

Recently intensive reasearch has been developed for enhancing the application of the FDTD method in time-domain electromagnetic field analysis concerning its storage and computation time requirements as well as the radiation boundary conditions for open structures. On the other hand, the time-domain IE techniques, which offer the attractive possibility of decreasing the dimensions of the problem, have been practically applied only to 1-D problems because of their lower computational efficiency in the case of complicated shapes of the boundaries and difficulties arising when imposing the bounadry conditions at dielectric-to-dielectric interfaces. Besides, the IE transient analysis has always been restricted to scattering or radiation problems.

The above mentioned shortcomings of the time-domain IE techniques can be overcome for the case of layered structures which are simple from geometrical point of view and are of great practical importance. In addition, the representation of field vectors by equivalent source currents and charges at the boundaries of interest offers the possibility for easy application of radiation boundary conditions. In this work the application of these techniques has been investigated and an efficient algorithm developed. The basic principles of the approach will be illustrated by a simple open microstrip structure (Fig.1).

THEORY

1. Basic equations.

The MFIE is applied to determine the boundary values of the field in two different regions (1 and 2, see Fig.1) with parameters (μ_0, ε_1) and (μ_0, ε_2) respectively. The time-domain boundary MFIE for i-th region in its most general form is:

$$\Theta_0 \vec{H}^{(i)} = \varepsilon_i \vec{I}_k^i\{-\vec{K}_s^{(i)}\} + \vec{I}_j^i\{\vec{J}_s^{(i)}\} + \frac{1}{\mu_0}\vec{I}_m^i\{m_s^{(i)}\} + \quad (1)$$
$$+ \quad \varepsilon_i \vec{I}_k^i\{(\hat{n}\times\vec{E}^{(i)})\} + \vec{I}_j^i\{(\hat{n}\times\vec{H}^{(i)})\} + \vec{I}_m^i\{(\hat{n}\vec{H}^{(i)})\}$$

where:
Θ_0 - angle which opens from the observation point into the region enclosed by the surface S,
$\vec{K}_s$,$\vec{J}_s$ and m_s - field source's magnetic currents, electric currents and equivalent magnetic charges respectively,
$\hat{n}$ - inward normal of the integrated surface at the point of integration.
$\vec{I}_k^i$,$\vec{I}_j^i$,$\vec{I}_m^i$ are vector integral operators as follows:

$$\vec{I}_k^i\{\vec{\xi}\} = \oint_s \frac{1}{r}\frac{\partial\vec{\xi}}{\partial\tau}\, ds,$$
$$\vec{I}_j^i\{\vec{\xi}\} = \oint_s \left(\frac{\vec{\xi}}{r^2} + \frac{1}{v_i r}\frac{\partial\vec{\xi}}{\partial\tau}\right) \times \hat{r}\, ds,$$
$$\vec{I}_m^i\{m\} = \oint_s \left(\frac{1}{r^2}m + \frac{1}{v_i r}\frac{\partial m}{\partial\tau}\right) \hat{r}\, ds.$$

Here:
r - distance between the observation point P and the integration point Q,
$\hat{r}$ - unit vector pointing from Q to P,
v_i - speed of light for the medium enclosed by the surface.

The surface integral denoted by $\oint$ is singular at the point of observation and for surfaces at which there are no sources it should be denoted by $⨏$ - the Cauchy principal value of the boundary integral from which the point of observation has been excluded. All field quantities are functions of both time and space. The dependence on

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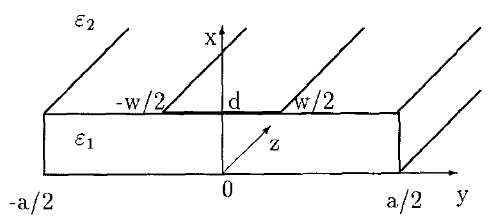

Figure 1: Open microstrip transmission line

time is represented by the retarded time $\tau = r - r/v$. Here the derivative $\partial/\partial\tau$ denotes the time derivative $\partial/\partial t$ for $t = \tau$.

In order to introduce equivalent currents and charges at the surfaces different from the source ones and obtain the integral equation relating them the cross product of all quantities with $\hat{n}_0$ is taken, where $\hat{n}_0$ is the inward surface normal at the observation point. The equivalent surface currents and charges are defined via the boundary relations:

$$\begin{aligned} \hat{n} \times \vec{E} &= -\vec{K} \\ \hat{n} \times \vec{H} &= \vec{J} \end{aligned} \tag{2}$$

and

$$\frac{\partial}{\partial t}(\hat{n}\vec{H}) = \frac{1}{\mu}\nabla_s.(\hat{n} \times \vec{E}) = -\frac{1}{\mu}\nabla_s.\vec{K} = \frac{1}{\mu}\frac{\partial m}{\partial t} \tag{3}$$

Thus, the MFIE for the i-th region is transformed into:

$$\begin{aligned} \Theta_0 \vec{J}^{(i)}(P) = \hat{n}_0 \times \Big(\varepsilon_i \vec{I}_k^i\{-\vec{K}_s^{(i)}\} + \vec{I}_j^i\{\vec{J}_s^{(i)}\} + \\ + \frac{1}{\mu_0}\vec{I}_m^i\{m_s^{(i)}\}\Big) + \\ + \hat{n}_0 \times \left(\varepsilon_i \vec{I}_k^i\{-\vec{K}^{(i)}\} + \vec{I}_j^i\{\vec{J}^{(i)}\} + \frac{1}{\mu_0}\vec{I}_m^i\{m^{(i)}\}\right) \end{aligned} \tag{4}$$

Equation (4) is applied to both regions of the example structure shown on Fig.1 - region (1) being the dielectric slab and region (2) - the air region. The boundary surfaces consist of source surfaces, the dielectric interface and outward radiating surfaces. The equivalence of sources and field surface currents/charges is obvious from (4). It implies that outward radiation from a surface segment would lead to zero contribution of that segment to the current estimated at any point in the region of interest. Thus, the boundaries subject to discretization and numerical evaluation are only the bottom ground conducting plane and the dielectric-to-air interface plane.

The unknown quantities at the bottom plane are both surface electric current components (J_y,J_z). Same are the unknowns of the strip. At the dielectric interface the unknowns are the two components of the surface electric currents as well as those of the surface magnetic currents (K_y,K_z) and the surface magnetic charges m for each region. The magnetic charges actually are not separate unknowns because of their relation to the magnetic currents via the boundary relation (3). Their values are stored and updated according to the relation:

$$m(t_k) = -\int_0^{t_k} \nabla_s.\vec{K}dt = m(t_{k-1}) - \int_{t_{k-1}}^{t_k} \nabla_s.\vec{K}dt \tag{5}$$

Here the $\nabla_s.\vec{K}$ is approximated with finite differences. For the edge patches where the integrated planes are truncated by radiation walls extrapolation formulas are applied.

The problem is defined by the two MFIE for both regions and the continuity boundary conditions at the dielectric interface which allow us to decrease the unknowns only to those of region (1):

$$\begin{aligned} \vec{K}^{(1)} &= -\vec{K}^{(2)} \\ \vec{E}^{(1)} &= -\vec{E}^{(2)} \\ m^{(1)} &= -m^{(2)} \end{aligned} \tag{6}$$

The procedure of solving both equations at the interface plane in general is complicated because it leads to coupling of both electric and magnetic currents. But in the case of layered structures electric and magnetic currents appear to be decoupled because of the zero value of the cross product $\hat{n}_0 \times (\vec{J} \times \hat{r})$ when the observation and integration points lie on a same plane (see equation (4)). Thus, after applying the boundary conditions (6) the electric current at the left side of the resultant equation becomes zero and the resultant equation contains only magnetic currents and magnetic charges in the surface integral of the interface:

$$\begin{aligned} 0 = SC^{(1)} - SC^{(2)} + CC^{(1)} - CC^{(2)} + \\ + \hat{n}_0 \times \left(\varepsilon_1 \vec{I}_k^1\{\vec{K}^{(1)}\} + \varepsilon_2 \vec{I}_k^2\{\vec{K}^{(1)}\}\right) + \\ + \hat{n}_0 \times \frac{1}{\mu_0}\left(\vec{I}_m^1\{m^{(1)}\} + \vec{I}_m^2\{m^{(1)}\}\right) \end{aligned} \tag{7}$$

where:
$SC^{(i)}$ - source contribution in the MFIE of region (i),
$CC^{(i)}$ - contribution of conducting surfaces other than the ones lying on the interface for region (i).

The equations for each component arising from (7) are coupled through the magnetic charges m which are functions of both components of $\vec{K}$. Therefore, they have to be solved as a system. After applying boundary discretization and point-matching procedure two coupled sets of equations are obtained:

$$\begin{aligned} \boldsymbol{ZZ} * \boldsymbol{K}_z + \boldsymbol{ZY} * \boldsymbol{K}_y &= \boldsymbol{FZ} \\ \boldsymbol{YZ} * \boldsymbol{K}_z + \boldsymbol{YY} * \boldsymbol{K}_y &= \boldsymbol{FY}, \end{aligned} \tag{8}$$

where the vectors $\boldsymbol{FZ}$ and $\boldsymbol{FY}$ are obtained after calculating the retarded-time contribution of all surfaces in

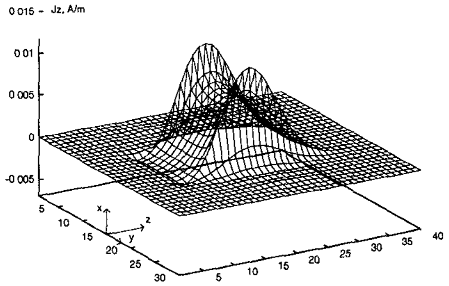

Figure 2: Longitudinal component of electric current J_z,$t = 90\Delta t$

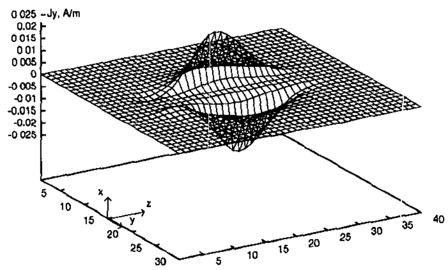

Figure 3: Transverse component of electric current J_y,$t = 90\Delta t$

the MFIE for J_z and J_y respectively. The matrices $\boldsymbol{ZZ}$, $\boldsymbol{ZY}$, $\boldsymbol{YZ}$, $\boldsymbol{YY}$ are obtained after discretization of the interface region and applying point-matching procedure. These matrices depend only on the geometry of the interface and are the same for every time step. Consequently, the elements of the overall matrix are first calculated, then the inverse matrix is found.

2. Discretization and basis functions.
Lagrange interpolation polynomials are used in time which is equivalent to an expansion in Taylor series up to the second order member in time:

$$f(\vec{x}_Q, \tau) = f(\vec{x}_Q, t_k) + \frac{\partial f(\vec{x}_Q, t_k)}{\partial t}\Delta\tau + \frac{1}{2}\frac{\partial^2 f(\vec{x}_Q, t_k)}{\partial t^2}\Delta\tau^2, \tag{9}$$

where:
$\Delta\tau = \tau - t_k = t_k - (t_0 - r/v)$,
t_0 being the current moment of time, t_k - the center of time interval for interpolation and $r = \mid \vec{r} \mid = \mid \vec{x}_P - \vec{x}_Q \mid$. All field quantities are considered constant in a single patch, i.e step basis functions are used in space. This approximation in time is desirable since it leads to integrals which have analytical solutions and are fast calculated during the time-stepping procedure. Smoother basis functions in space are desirable only in the case of complex geometry when the integrals including charges may have high order singularities. For the considered case of layered structures with the Lagrange interpolation in time this is not needed. Discretization steps in space (patch size) and time are related by $\Delta h = c\Delta t$ where $c = v_2$ is the velocity of light.

Special care should be taken for the self-patch integration of magnetic charges which gradient cannot be neglected in this case. Their expansion in Taylor series now is:

$$\begin{aligned} m(\vec{x}_Q, \tau) &= m(\vec{x}_Q, t_k) - \vec{r}\nabla_s m(\vec{x}_Q, t_0) \\ &- \frac{r}{v}\frac{\partial m(\vec{x}_Q, t_0)}{\partial t} + \frac{1}{2}\left(\frac{r}{v}\right)^2 \frac{\partial^2 m(\vec{x}_Q, t_0)}{\partial t^2} \end{aligned} \tag{10}$$

Since charges are determined according to (5) equations obtained from (7) are solved as a system (see 8). Matrices involved are sparce and matrix inversion is made just once since geometry does not change in time.

RESULTS AND DISCUSSION

The resources required by this approach in respect with memory and computation time depend not only on the size of the structure (number of patches to be integrated) but also on the ratio of contuctor patches to dielectric interface patches. At every dielectric interface patch five quantities are stored (the two components of J and K and the magnetic charge m), whereas at every conductor patch only two quantities are needed (both components of J). Besides, it is necessary to store a history package of values back to a retarded time corresponding to the largest dimension of the structure. Thus, if the number of patches in the y direction is denoted by A for the grounded plane, by W for the strip and by L in the direction of z-axis, then at every time step

$$N = (7A - 3W) \times L$$

unknowns are to be calculated and stored. The dimension of the history package depends mainly on the largest dimension ($\approx L$) and the resonant properties of the structure. The number of space steps along x-axis is of no significant importance unless it is comparable with L which is not the case with the layered structures in practice.

There is no need of numerical integration when Lagrange interpolation in time is used. The integration of every patch is reduced to six basic integrals of local coordinates, namely:

$$I_r = \iint_{-1/2}^{1/2} \frac{1}{\sqrt{(\xi + \Delta n_i)^2 + (\eta + \Delta n_j)^2}} d\xi d\eta$$

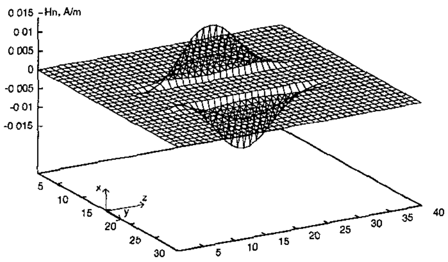

Figure 4: Normal component of magnetic field H_x,$t = 90\Delta t$

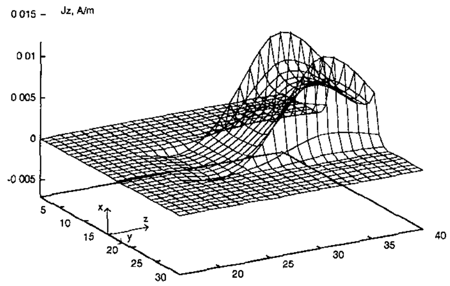

Figure 5: Longitudinal component of electric current J_z,$t = 130\Delta t$

$$
\begin{aligned}
I_{r^3} &= \iint_{-1/2}^{1/2} \frac{1}{\left(\sqrt{(\xi+\Delta n_i)^2+(\eta+\Delta n_j)^2}\right)^3}\, d\xi d\eta \\
I_r^{\xi} &= \iint_{-1/2}^{1/2} \frac{(\xi+\Delta n_i)}{\sqrt{(\xi+\Delta n_i)^2+(\eta+\Delta n_j)^2}}\, d\xi d\eta \\
I_r^{\eta} &= \iint_{-1/2}^{1/2} \frac{(\eta+\Delta n_j)}{\sqrt{(\xi+\Delta n_i)^2+(\eta+\Delta n_j)^2}}\, d\xi d\eta \\
I_{r^3}^{\xi} &= \iint_{-1/2}^{1/2} \frac{(\xi+\Delta n_j)}{\left(\sqrt{(\xi+\Delta n_i)^2+(\eta+\Delta n_j)^2}\right)^3}\, d\xi d\eta \\
I_{r^3}^{\eta} &= \iint_{-1/2}^{1/2} \frac{(\eta+\Delta n_j)}{\left(\sqrt{(\xi+\Delta n_i)^2+(\eta+\Delta n_j)^2}\right)^3}\, d\xi d\eta
\end{aligned}
$$

which have analytical solution. Here Δn_i is the number of space steps with respect to the i-axis between the integration point and the observation point.

Numerical simulation was carried out for the microstrip line in Fig.1. with number of space steps $A = 30$ in the y-direction ($W = 6$ for the strip conductor) and $L = 40$ in the z-direction. The number of steps in the x-direction is $B = 6$. Here the space step is $\Delta h = 0.1\ mm$. The dielectric constant of the substrate is $\varepsilon_r = 9.6$. Gaussian pulse excitation is used at $z = 0$, where the $\vec{E}$ field and $\vec{H}$ field are replaced by equivalent current sources, the $\vec{J}^i$ having only x-component and $\vec{K}^i$ - only y-component. Constant distribution of the source currents is assumed for $0 < x < d$ and $-w/2 < y < w/2$ and zero elsewhere. The source field is considered to be TEM, so, no magnetic-charge sources are present. The pulse width (from maximum value to cut point) is assumed $\beta = 20$. Fig.2 shows the longitudinal J_z component (H_y-component) at time step $90\Delta t$. Fig.3 and Fig.4 show the J_y (H_z) component and the H_x component (m/μ_0) respectively at the same time reference. Fig.5 shows the J_z component at a time-step $130\Delta t$.

CONCLUSION

A new possibility for analyzing transient fields in layered structures is proposed in this paper. A numerical approach for coupling the MFIE on mixed conductor and dielectric interfaces has been developed. It has been shown that the method is especially efficient for radiating and open boundary problems.

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REFERENCES

[1] K. Kunz and R.Luebbers, Finite-Difference Time-Domain Method for Electromagnetics, CRC Press, 1993

[2] L.B.Felsen ed., Transient Electromagnetic Fields, Springer - Verlag, 1976

[3] R.Mittra ed., Computer Techniques for Electromagnetics, Pergamon Press, 1973

[4] Egon Marx, Integral equation for scattering by a dielectric, IEEE Trans. AP, vol. AP-32, No 2, Feb. 1984

[5] Egon Marx, Self-patch integrals in transient electromagnetic scattering, IEEE Trans. AP, vol. AP-33, No 7, July 1985